EFFECT OF WALL BUBBLES ON THE EXCITATION OF THERMOACOUSTIC OSCILLATIONS IN BOILING FLOWS

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It is shown that wall vapor bubbles can affect the excitation of thermoacoustic oscillations only within narrow ranges of the regime parameters for boiling in short channels.

Thermoacoustic oscillations (TAO) develop during surface boiling in heated channels [1-4]. Aspects of the description of the mechanisms responsible for the occurrence of these oscillations have been the subject of some debate. Most authors [3-6] believe that the oscillations are due to an interaction between wall vapor bubbles and acoustic perturbations in the flow. However, the arguments made in support of this proposition are of a qualitative nature and are to a large extent based on intuitive considerations.

In the present study, we attempt to analyze the contribution of wall vapor to the excitation of thermoacoustic oscillations in boiling flows. We will not examine the effect of bubbles located in the bulk flow. Following [7], we can write an equation which describes the change in the total energy of the acoustic oscillations E in the flow per unit length:

$$\frac{\partial E}{\partial t} = \Pi A_F , \tag{1}$$

where A_F is the power of the surface sources of acoustic energy, averaged over the period T, per unit surface:

$$A_F = \frac{1}{T}\int_0^T \delta P \delta w_F \, dt \tag{2}$$

(δw_F is the mean velocity of the wall phase boundary). In subsequent discussions, we will proceed on the basis of the fact that, in the general case, bubbles of different sizes are located on the unit surface at an arbitrary moment of time. Differences in the sizes of the wall bubbles at the moments of time t are due to differences in their lifetime τ_i up to this moment. Since conditions under which $\tau_i \ll T$ are possible in the general case, we will examine a certain wall volume of vapor on a unit surface:

$$V_{wa}(t) = \sum_{i=1}^{n} V_i(t, \tau_i), \tag{3}$$

where n is the number of bubbles acting on the unit surface at the moment of time t. In contrast to the case of individual bubbles, the lifetime of the wall volume V_{wa} can be assumed to be greater than the period of the oscillations T. Then the quantity dV_{wa}/dt characterizes the mean interphase velocity over the surface at the boundary between the flow and the wall vapor, i.e.

$$\delta w_F \approx d(\delta V_{wa})/dt. \tag{4}$$

With allowance for (4), Eq. (2) takes the form

$$A_F = \frac{1}{T}\int_0^T \delta P \frac{d(\delta V_{wa})}{dt}\, dt. \tag{5}$$

At $A_F > 0$, thermoacoustic oscillations will be generated in the channel as a result of the behavior of the wall bubbles. At $A_F < 0$, the process stabilizes. At $A_F = 0$, the wall vapor is "neutral" relative to the excitation of oscillations. In fact, $A_F T$ is the work done by the vapor bubbles on a unit surface of the channel, averaged over the period of oscillation

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in the acoustic wave. It should be noted that, with allowance for (5), Eq. (1) is the analog of the familiar Rayleigh criterion [7] for the case in question.

The complexity of describing the generation and growth of bubbles during surface boiling is too great to permit a satisfactorily exact solution. Thus, taking into account the qualitative nature of the proposed analysis, we will make use of the following basic assumptions.

1. We will examine the case when the bubbles separate from the channel surface. According to [4, 5], the following can be approximately taken for the stage of the growth of wall bubbles

$$R = \beta_g t. \tag{6}$$

2. The behavior of the bubbles at the moment of time t is affected by the history of the process in the acoustic wave, beginning with the moment $t - (\tau_i + \vartheta_2)$. The effect of changes in the external conditions at a previous time on the process of vapor formation at the moment of time t will be ignored.

3. The waiting period ϑ_2 before the appearance of a bubble is determined mainly by the separation size of the previous bubble. Here, a reduction in pressure and, accordingly, an increase in d_{sp} lead to an increase in ϑ_2 [8-10]:

$$\partial\vartheta_2/\partial P < 0. \tag{7}$$

4. The frequency of bubble separation f depends slightly on the pressure in the acoustic wave. We will ignore perturbations of the second or higher order of smallness. We write the heat-balance equation for the wall volume of vapor in the unperturbed state in the form

$$r\frac{dV_{wa}\rho''}{dt} = \sum_{i=1}^{n} F_i(t,\ t_{0i})\, q_i(t,\ t_{0i}) - n(t-\vartheta_1)\, f V_{sp}(t)\, \rho'' r. \tag{8}$$

The first term in the right side of (8) reflects the change in the mass of the wall vapor as a result of interphase mass transfer, while the second term reflects the reduction in volume as a result of the separation of bubbles. Expressed in terms of the perturbations, Eq. (8) appears as follows with allowance for the assumptions made above:

$$\frac{d\delta V_{wa}}{dt} = \sum_{i=1}^{n}\left[\frac{\partial F_i}{\partial V_i}\frac{q_i}{r}\delta\bar{V}_i + \frac{F_i}{r}\delta q_i\right] - V_{wa}\frac{d\rho''}{dP}\frac{d\delta P}{dt} - V_{sp}(t)\frac{\partial nf}{\partial P}\delta P(t-\vartheta_1) - nf\delta V_{sp}(t). \tag{9}$$

Perturbations of the volume of an individual bubble $\delta\bar{V}_i$ are caused both by a perturbation of acoustic pressure during the life of the bubble τ_i and by a time shift in the beginning of bubble growth t_{0i} under the influence of acoustic disturbances. Here, perturbations of the quantity t_{0i} are caused by disturbances occurring both at the moment of separation of the previous bubble $t - \tau_i - \vartheta_2$ and just before the moment of the beginning of bubble growth t_{0i}. Then

$$\delta\bar{V}_i = \delta V_i(\delta P) + \frac{\partial V_i}{\partial t}\left|\frac{\partial\tau_i}{\partial\vartheta_2}\right|\left|\frac{\partial\vartheta_2}{\partial P}\right|\delta P(t-\tau_i-\vartheta_2) - \frac{\partial V_i}{\partial t}\left|\frac{\partial t_{0i}}{\partial P}\right|\delta P(t-\tau_i). \tag{10}$$

With allowance for (10), Eq. (9) is changed to the form:

$$\begin{aligned}\frac{d\delta V_{wa}}{dt} = \sum_{i=1}^{n}\Bigg\{\frac{1}{r}\frac{\partial F_i}{\partial V_i}q_i\Bigg[\delta V_i(t) + \frac{\partial V_i}{\partial t}\left|\frac{\partial\tau_{0i}}{\partial\vartheta_2}\right|\left|\frac{\partial\vartheta_2}{\partial P}\right|\delta P(t-\vartheta_2 -\\ -\tau_i) - \frac{\partial V_i}{\partial t}\left|\frac{\partial\tau_i}{\partial P}\right|\delta P(t-\tau_i)\Bigg] + \frac{F_i}{r}\delta q_i(t)\Bigg\} - V_{wa}\frac{d\rho''}{dP}\frac{d\delta P}{dt} -\\ - V_{sp}(t)\frac{\partial nf}{\partial P}\delta P(t-\vartheta_2) + nf\left|\frac{\partial V_{sp}}{\partial P}\right|\delta P(t).\end{aligned} \tag{11}$$

Pressure perturbations in the acoustic wave change in accordance with a harmonic law ($a_g > 0$)

$$\delta P(t) = a_g \sin \omega t. \qquad (12)$$

Let us determine the effect of each term in the right side of (11) on the value of A_F in accordance with (5). With allowance for (11), we represent the integral (5) in the form

$$A_F = A_{F_1} + A_{F_2} + A_{F_3} + \cdots + A_{F_7}, \qquad (13)$$

where

$$A_{F_1} = \frac{1}{Tr}\int_0^T \delta P(t)\left(\sum_{i=1}^{n} \frac{\partial F_i}{\partial V_i} q_i \delta V_i\right) dt, \qquad (14)$$

$$A_{F_2} = \frac{1}{Tr}\int_0^T \delta P(t)\left[\frac{\partial F_i}{\partial V_i}\frac{\partial V_i}{\partial t}\left|\frac{\partial \tau_i}{\partial \vartheta_2}\right|\left|\frac{\partial \vartheta_2}{\partial P}\right| q_i \delta P(t - \vartheta_2 - \tau_i)\right] dt; \qquad (15)$$

$$A_{F_3} = \frac{1}{Tr}\int_0^T \delta P(t)\left[-\sum_{i=1}^{n} \frac{\partial V_i}{\partial t}\left|\frac{\partial \tau_i}{\partial P}\right|\frac{\partial F_i}{\partial V_i} q_i \delta P(t - \tau_i)\right] dt \qquad (16)$$

$$A_{F_4} = \frac{1}{Tr}\int_0^T \delta P(t)\left(\sum_{i=1}^{n} F_i \delta q_i\right) dt; \qquad (17)$$

$$A_{F_5} = \frac{1}{T\rho''} V_{wa} \frac{d\rho''}{dP}\int_0^T \delta P(t)\,[d(\delta P)/dt]\, dt; \qquad (18)$$

$$A_{F_6} = -\frac{1}{T}\frac{\partial nf}{\partial P} V_{sp}\int_0^T \delta P(t - \vartheta_2)\,\delta P(t)\, dt; \qquad (19)$$

$$A_{F_7} = -\frac{1}{T} nf\left|\frac{\partial V_{sp}}{\partial P}\right|\int_0^T \delta^2 P(t)\, dt. \qquad (20)$$

With allowance for (12)

$$A_{F_7} = \frac{nf}{T}\left|\frac{\partial V_{sp}}{\partial P}\right| a_g^2 \int_0^T \sin^2 \omega t\, \partial t > 0. \qquad (21)$$

Thus, the given term, reflecting the effect of perturbations in the acoustic wave on bubble dimensions at the moment of separation, is capable of causing a build-up of the oscillations. The essence of this effect is as follows. The separation dimensions decrease with an increase in the pressure caused by the acoustic wave. Thus, the vapor in the wall layer remains more abundant ("vapor inflow") than in the undisturbed state. Conversely, the amount of vapor in the wall layer is less than the amount present in the unperturbed state when pressure in the acoustic wave decreases. In accordance with the well-known Rayleigh criterion [7], a simultaneous increase (decrease) in pressure and mass should lead to the excitation of oscillations in the wall region. However, it must be kept in mind that the change in the "vapor inflow" ("vapor outflow") in the wall region which occurs during bubble separation corresponds to the opposing "vapor outflow" ("vapor inflow") in the bulk flow at this moment. Thus, on the whole, the effect of A_{F_7} on the build-up of oscillations in the flow is neutral. The influence of the term A_{F_6} is similar. This term reflects the effect of the corrected number of separating bubbles on the build-up of oscillations in the flow. The term A_{F_5} accounts for the effect of the compressibility of the vapor on the development of oscillations. With allowance for (12), it follows from (18) that $A_{F_5} = 0$, i.e., its effect is also neutral. The third term, A_{F_3}, reflects the effect of a change in the dimensions of the bubbles on the excitation of oscillations under the influence of the acoustic wave at the onset of the bubbles' nucleation. With allowance for (12), the expression for this term has the form

$$A_{F_3} = -\frac{1}{Tr}\sum_{i=1}^{n} a_g^2\left|\frac{\partial \tau_i}{\partial P}\right|\int_0^T \frac{\partial V_i}{\partial t}\frac{\partial F_i}{\partial V_i} q_i \sin\omega(t - \tau_i)\sin\omega t\, dt. \qquad (22)$$

Using (6) for the undisturbed state and assuming that $q_i \approx r\rho'' dR_i/dt$, we find from Eq. (22) that

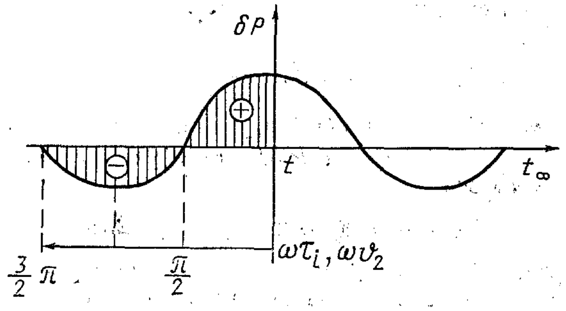

Fig. 1. Mechanism of excitation of oscillations by wall bubbles.

$$A_{F_3} = -\sum_{i=1}^{n} |a_0| T^2 (\cos \omega\tau_i + \sin \omega\tau_i/2\pi), \tag{23}$$

where $|a_0| = 2\pi a_g^2 |\partial\tau_i/\partial P| \beta_g^3 r\rho'' T^{-1}$. Thus, the term in question will promote the development of oscillations approximately as

$$\frac{\pi}{2}(4k+1) < \omega\tau_i < \frac{3\pi}{2}(4k+3); \; k = 0, 1, 2, \ldots \tag{24}$$

The result obtained here can be represented physically as follows. Let a pressure disturbance increase at the moment t, $\delta P(t) > 0$ (see Fig. 1), and let condition (24) be satisfied. Then the pressure disturbance will be negative at the onset of bubble growth. This will lead to earlier nucleation of bubbles compared to the undisturbed state. Thus, at the moment t, there is an increase in the surface and the rate of supply of vapor relative to the undisturbed state. In accordance with the Rayleigh criterion, the phase coincidence of the increases in pressure and the rate of mass inflow leads to the development of oscillations. Similar conclusions follow when we examine the decrease in the pressure disturbance at the moment t and the satisfaction of conditions (24). If the density of the vaporization centers is sufficiently great (it can be assumed at any moment of time that there are an approximately equal number of centers satisfying and not satisfying condition (24), then the above-examined term A_{F_3} is neutral in relation to the excitation of oscillations.

Similarly, by transforming A_{F_2} we obtain

$$A_{F_2} = \sum_{i=1}^{n} |b_0| T^2 [\cos \omega(\vartheta_2 + \tau_i) + \sin \omega(\vartheta_2 + \tau_i)/2\pi], \tag{25}$$

where $|b_0| = 2\pi a_g^2 \left|\frac{\partial\tau_i}{\partial\vartheta_2}\right| \left|\frac{\partial\vartheta_2}{\partial P}\right| \beta_g^3 r\rho'' T^{-1}$. Thus, in accordance with (25), oscillations will be excited approximately when

$$0 < \omega(\vartheta_2 + \tau_i) < \frac{\pi}{2} \text{ or } \pi\left(\frac{3}{2} + 2k\right) < \omega(\vartheta_2 + \tau_i) < \pi\left(\frac{5}{2} + 2k\right). \tag{26}$$

Let us examine the physical essence of condition (26) in special cases. Let $\vartheta_2 \gg \tau_i$. These conditions are typical of a region of low pressures and high degrees of subcooling. Then at the moment t $\delta P(t) > 0 (<0)$, while at the moment of separation of the previous bubbles the pressure disturbance of the acoustic wave is also positive (or negative), in accordance with (26) (see Fig. 1). Thus, at the moment of separation, the size of the bubbles will be less (greater) relative to the undisturbed state. The time ϑ_2 also decreases (increases), which leads to an increase (decrease) in the time of bubble growth τ_i and, accordingly, to an increase (decrease) in the bubbles' surface and the rate of vapor inflow relative to the undisturbed state. Consequently, with the satisfaction of conditions (26) and $\vartheta_2 \gg \tau_i$, there is a simultaneous increase (or decrease) in the pressure disturbances and the "vapor inflow." In accordance with the Rayleigh criterion, such a development promotes the growth of oscillations. Let us examine the other limiting case (26), when $\vartheta_2 \ll \tau_i$. These conditions are characteristic of high pressures and low degrees of subcooling in the flow. In this case, as follows from (26), the conditions for the excitation of oscillations are opposite to condition (24):

$$\cos\omega\tau_i + \sin\omega\tau_i/(2\pi) > 0. \quad (27)$$

In fact, as indicated above, relation (24) reflects the conditions for the excitation of oscillations as a result of perturbations of the time of the onset of bubble growth, while relation (27) reflects the conditions for the excitation of oscillations due to perturbations of the waiting time ϑ_2. With (24), oscillations will on the one hand develop as a result of a certain disturbance of the acoustic wave at the moment t_{0i} (see above) and on the other hand be discouraged from developing by the mechanism of the perturbation of ϑ_2.

Let us examine the effect of the term A_{F_1} on the excitation of oscillations due to a change in the dimensions of the bubbles during their growth in the acoustic wave. In the general case, it is very difficult to describe the quantity δV_i in (14) because of the appreciable nonuniformity of the temperature field near the bubble, inertial efffects, and other factors. Considering the qualitative character of the proposed analysis, to approximately describe δV_i under the given conditions we will make use of the results in [5]:

$$\delta R(t) \simeq B[\cos(\omega t + \gamma_1) - \cos\gamma_1], \quad (28)$$

where $B > 0$; $\gamma_1 = \mathrm{arctg}\,[(1-\mu_\omega)/(1+\mu_\omega)]$; $\mu_\omega = \sin\bar{\omega}\cos\omega/\mathrm{ch}\,\bar{\omega}\,\mathrm{sh}\,\bar{\omega}$; $\bar{\omega} = \delta_t\sqrt{\omega/2a_L}$. Since $\delta V(t) = 4\pi R^2\delta R$, then with allowance for (6), (12), and (28), Eq. (14) takes the form

$$A_{F_1} = -T^2|C_0|[\sin\gamma_1 + \cos\gamma_1/(2\pi)], \quad (29)$$

where $|c_0| = 2\pi r\rho'' na_g BT^{-1}$. Numerical analysis of the term in the brackets shows that it is positive throughout the investigated range of variation of the frequencies. Thus, $A_{F_1} < 0$. As a result, in accordance with (29), we conclude that the term A_{F_1} impedes the generation of thermoacoustic oscillations.

Finally, let us examine the term A_{F_4}. The effect of this term on the generation of oscillations is due to a change in interphase heat transfer resulting from perturbations of the acoustic wave. Following [5], we will write the boundary-value problem for the growth of a wall bubble (without allowance for inertial effects) in the following approximate form:

$$\frac{\partial T_L}{\partial t} = a_L\frac{\partial^2 T_L}{\partial y^2};\; q = -\lambda_L\left(\frac{\partial T_L}{\partial y}\right)_{y=0}; \quad (30)$$

$$\left(\frac{\partial T_L}{\partial y}\right)_{t=0} = -\frac{T_{wa} - T_{L\infty}}{\delta_t};\quad T_L(R, t) = T_{s0} + \frac{\partial T_s}{\partial P}\delta P(t);$$
$$T_L(\delta_t, R) = T_{L\infty}. \quad (31)$$

Ignoring initial perturbations of the temperature of the liquid and allowing for (12), we obtain the following familiar solution in perturbations from (30-31)

$$\delta q = -\lambda_L\sqrt{\frac{\omega}{a_L}}\frac{dT_s}{dP}a_g\sin(\omega t + \gamma_2),\ \text{where}\ \gamma_2 \approx \pi/6. \quad (32)$$

Then performing some cumbersome transformations with allowance for (12), we find from (17) that

$$A_{F_4} = -T^3|d_0|\left[\cos\gamma_2\int_0^1 \xi^2\sin^2(2\pi\xi)\,d\xi - \frac{\sin\gamma_2}{8\pi}\right] < 0, \quad (33)$$

where $|d_0| = 4\pi\beta_g^2 na_g T^{-1}$. The physical significance of the result (33) is that an increase (decrease) in the pressure perturbation of the acoustic wave is accompanied by an increase (decrease) in T_s. Thus, there is a decrease (increase) in heat transfer between the growing bubble and the liquid. As a result, an increase (decrease) in pressure is accompanied, as a result of the above phenomenon, by a decrease (increase) in the rate of supply of vapor. In accordance with the Rayleigh criterion [7], such an event impedes the excitation of oscillations.

Thus, the effect of wall vapor bubbles on the excitation of thermoacoustic oscillations in a boiling flow is connected mainly with the existence of a certain relationship between the frequency of oscillations of the acoustic wave ω on the one hand and the time of growth τ_i (max $\tau_i = \vartheta_1$) and waiting time ϑ_2 for wall bubbles on the other hand. Relations (24) and (26) are approximate conditions for the excitation of oscillations. In these expressions, the quantities ϑ_1 and ϑ_2 depend on the specific regime parameters and the physico-chemical properties of the contacting phases. The author of [4] conducted experiments in which he obtained oscillograms of the pressure and wall volume of vapor during surface boiling. Analysis of these oscillograms by means of Eq. (5) makes it possible to conclude that $A_F < 0$ in the cases examined here. The tests in [4] were conducted at atmospheric pressure with large amounts of subcooling (tens of degrees). The conditions prevailing in these experiments corresponded to the case examined above: $\vartheta_1 \ll T$, $\vartheta_2 \sim T$, and it was necessary to use (26) to perform the analysis at $\vartheta_2 \gg \tau_i$. According to estimates obtained by using familiar expressions for ϑ_2 [8], conditions (26) were not realized under these conditions, i.e., the wall bubbles did negative work and prevented the excitation of thermoacoustic oscillations. Thus, the existence of oscillations in the given case was due to vapor bubbles moving in the flow, not "sitting" on the wall. For the case when $\vartheta_1, \vartheta_2 \ll T$ (fairly long channels), the wall volume V_{wa} will be a quasisteady function of pressure, i.e., will be independent of the history of the oscillation process and will be determined only by the perturbations at the moment t (see Fig. 1). Then, in accordance with (3): $\delta V_{wa} = (dV_{wa}/dP) \times \delta P(t)$. Then inserting this result into Eq. (5), we obtain: $A_F = 0$. Thus, in sufficiently long channels, wall bubbles will be neutral relative to the excitation of oscillations. In connection with this, mechanisms examined in [3, 4, 6] and based on the action of wall bubbles will not be decisive – at least in long channels, when $T \gg \vartheta_1, \vartheta_2$. It is known from experiments [1, 2] that thermoacoustic oscillations in heated channels are excited and develop mainly at $\phi > 0.1$. At such values of vapor content, the concentration of "bulk" bubbles in the cross section of the channel is considerably higher than the concentration of wall bubbles. Thus, wall bubbles can affect the development of thermoacoustic oscillations during boiling in short channels only within a narrow range of regime parameters.

NOTATION

V, volume; t, time; $\omega = 2\pi T$, cyclic frequency of perturbations; P, pressure; j, imaginary unit; ϑ_1, ϑ_2, mean lifetime and mean time before the appearance of wall bubbles for the specified regime parameters; n, mean density of vaporization centers on the heating surface for the specified regime parameters; δy, nonsteady perturbation of the parameter y; δ_t, thickness of the superheated layer near the heating surface; a_L, diffusivity of the liquid; Π, perimeter of the channel; β_g, model of bubble growth; ϕ, true volumetric vapor content; d_{sp}, V_{sp}, separation diameter of a bubble and its corresponding volume.

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